PHYSICS 2DL – SPRING 2010

MODERN PHYSICS LABORATORY

Monday April 26, 2010

Prof. Brian Keating

UCSD

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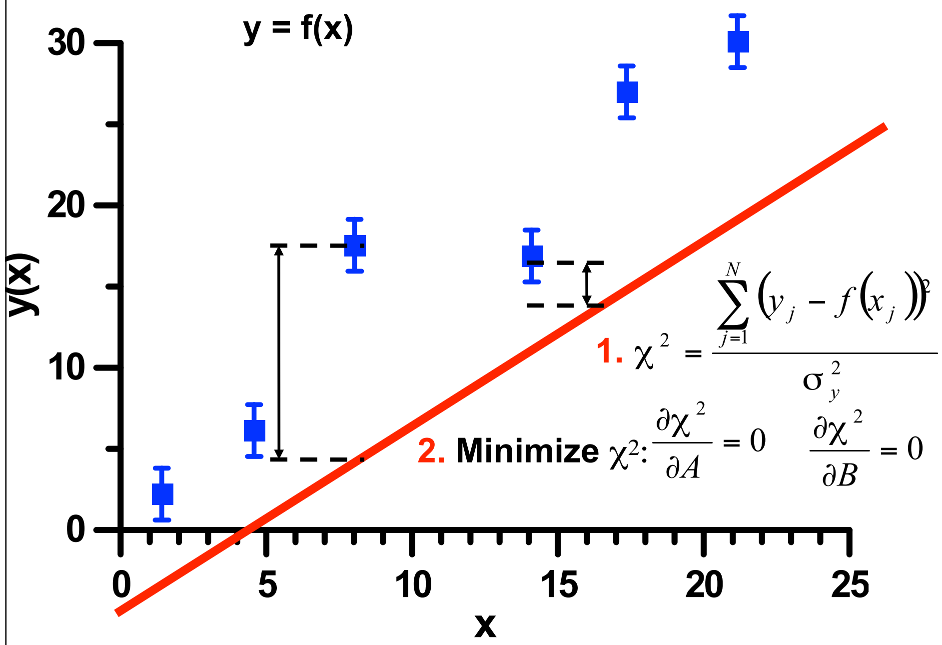
2DL More on the Labs

- Davisson-Germer Experiment
- e/m for the electron
- Optical Coherence
- Franck-Hertz experiment

Next time: LEAST SQUARES FITTING



Fitting Voltage Data to V=IR

$$\frac{\partial \chi^2}{\partial R} = 0$$

$$IMPLIES:$$

N = number of data points. In this example, N=4

$$R = \frac{\sum_i^N I_i V_i}{\sum_i^N I^2{}_i}$$

What is the Error on the Best-Fit Parameter R?

Our general formula, which always applies, is:

$$\sigma_R = \sqrt{\left(\frac{\partial R}{\partial V_1}\right)^2 \sigma_{V_1}^2 + \left(\frac{\partial R}{\partial V_2}\right)^2 \sigma_{V_2}^2 + \ldots \left(\frac{\partial R}{\partial V_N}\right)^2 \sigma_{V_N}^2}$$

Since:

$$\left(\frac{\partial R}{\partial V_1}\right)^2 = I_1^2, \left(\frac{\partial R}{\partial V_N}\right)^2 = I_N^2$$

$$and : \sigma_{V_N} = 1mV$$

Putting it all together:

$$so : \sigma_R = \frac{1mV\sqrt{\sum_i^N I_i^2}}{\sum_i^N I_i^2}$$

Check units are right, error has same units as R.

LEAST SQUARES FITTING EXAMPLE

current [mA]	voltage [mV]	voltage error [mV]	voltage measured [mV]	voltage uncertainty [mV]	x^2 [mA^2]	xy [mA*mV]	voltage from fit [mV]
1.0	2.0	-0.8	1.2	1.0	1.0	1.2	2.1
2.0	4.0	0.3	4.3	1.0	4.0	8.6	4.2
3.0	6.0	1.0	7.0	1.0	9.0	20.9	6.3
4.0	8.0	0.0	8.0	1.0	16.0	32.1	8.4
this is "x_i"			this is "y_i"	this is "σ_Y"	$\sum x^2$ 30.0	$\sum xy$ 62.9	

This is the true signal

This is the true signal with error (uncertainty).

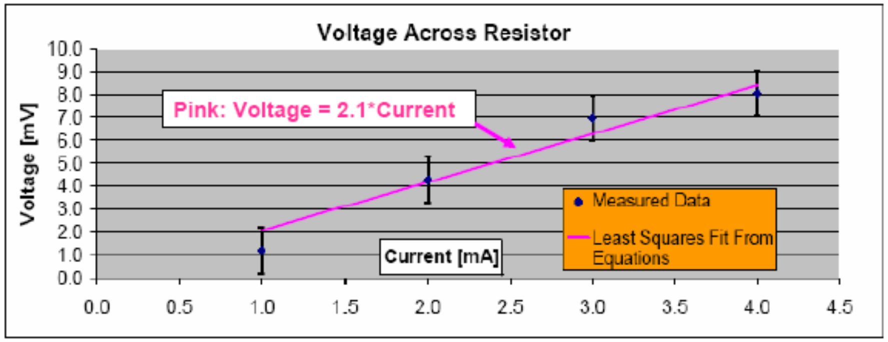
What we would measure in real-life

Our model: V=I×R

R from Fit: $R = \sum(xy)/\sum(x^2)$ 2.1 Ω

Error in R comes from partial derivative of numerator with respect to y_i only

Error in R $\sigma_R = \sigma_Y / \sqrt{(\sum x^2)}$ 0.2 Ω



Louis de Broglie

- **Approach: unify ideas of Planck and Einstein (light is quantized) with those of Bohr for the atom.**
  - **We know light is a wave (inteference effects) which sometimes acts like a particle (Planck's quanta, Einstein and the photoelectric effect).**
  - **If light (manifestly a wave) can sometimes be also viewed as a particle, why cannot electrons (manifestly a particle) be sometimes viewed as a wave?**
- **Additional motivation: Quantization rules occur naturally in waves. Perhaps Bohr's quantization rule might be understood in terms of "matter waves".**

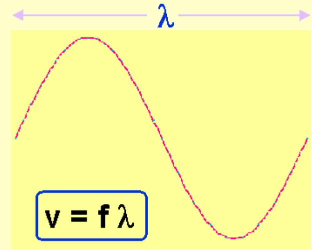
Waves and Quantization

- Recall in our earlier study of waves:

  λ = wavelength = distance it takes for pattern to repeat

  f = frequency = how many times a given point reaches maximum each second

  v = velocity of wave



- **Standing waves** as example of a "quantization rule":
  - Suppose both ends of a string are fixed so that they can't move.
  - For a fixed length of string, only waves with certain wavelengths will survive to make standing waves... those wavelengths which have zeroes at the ends of the string.

$L = \lambda / 2$, $L = \lambda$, etc. ➡ **Quantization rule:** $L = n \lambda / 2$, $n=1,2,\ldots$

The de Broglie Wavelength

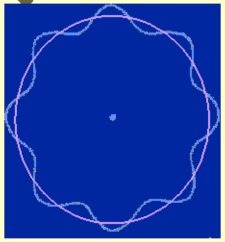
- **Big question:** How can we quantify deBroglie's hypothesis that matter can sometimes be viewed as waves? What is the wavelength of an electron?

- **de Broglie's idea:** define wavelength of electron so that same formula works for light also, when expressed in terms of momentum!
  - What is momentum of photon? This is known from relativity:
    - $p = E / c$ (plausible since: $E = mc^2$ and $p = mc \Rightarrow E = pc$)
  - How is momentum of photon related to its wavelength?
    - from photoelectric effect: $E = h\nu \Rightarrow pc = h\nu$
    - change frequency to wavelength: $c = \lambda\nu \Rightarrow c/\nu = \lambda$

$$\Rightarrow \boxed{p\,\lambda = h} \Rightarrow \boxed{\lambda = h / p}$$

de Broglie Waves & the Bohr Atom

- de Broglie's wave relation ($\lambda = h/p$) can now be used to "derive" Bohr's quantization rule for the hydrogen atom ($L = n\,(h/2\pi)$).
- How? if an electron is to be viewed as a wave whose wavelength is determined by its momentum, then in the H atom, the electron can have only certain momenta, namely those that correspond to the wavelengths of the standing waves on the orbit.



Standing wave condition:
Circumference of circle = integral number of wavelengths

$$2\pi R = n\lambda = n\,h/p$$

$$p R = n\,h/2\pi$$

$$\boxed{L = n\,h/2\pi}$$

Conclusion: Bohr's quantization rule is just the requirement that the electron wave be a standing wave on the circular orbit!

The Significance of $\lambda = h/p$

- The de Broglie wave hypothesis "explains" the previously arbitrary quantization rule of Bohr ($L=n(h/2\pi)$). But this hypothesis is not restricted to electrons in the Hydrogen atom! Can we find any more evidence for the wave nature of matter?
- Where to look? Interference phenomena:
  - The key property of waves is that they show interference. Recall the interference patterns made by visible light passing through two slits or sound from two speakers.
  - The problem with electrons - typical wavelengths are very small and one must find a way to observe the interference over very small distances

Davisson-Germer Experiment (1927)

- **Details:** Actual experiment involve electrons scattered from a Nickel crystal.
- **Done at Bell Labs** -- where Davisson and Germer studying electrons in vacuum tubes

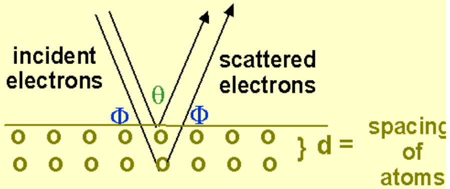
- For fixed energy of the incident electrons, (E = 54 eV, λ = 1.65Å), we expect to see an interference peak in the scattered electrons if the angle Φ is such that the path difference is an integral number of wavelengths.

- Alternatively, for a fixed scattering angle ($\Phi = 65°$), we expect to see the scattering rate to be large for incident electron energies which correspond to de Broglie wavelengths which are equal to the path difference between layers divided by an integer.

Davisson-Germer Experiment (1927)

- **Idea:** Interference effects can be seen by scattering electrons scattered from a crystal.

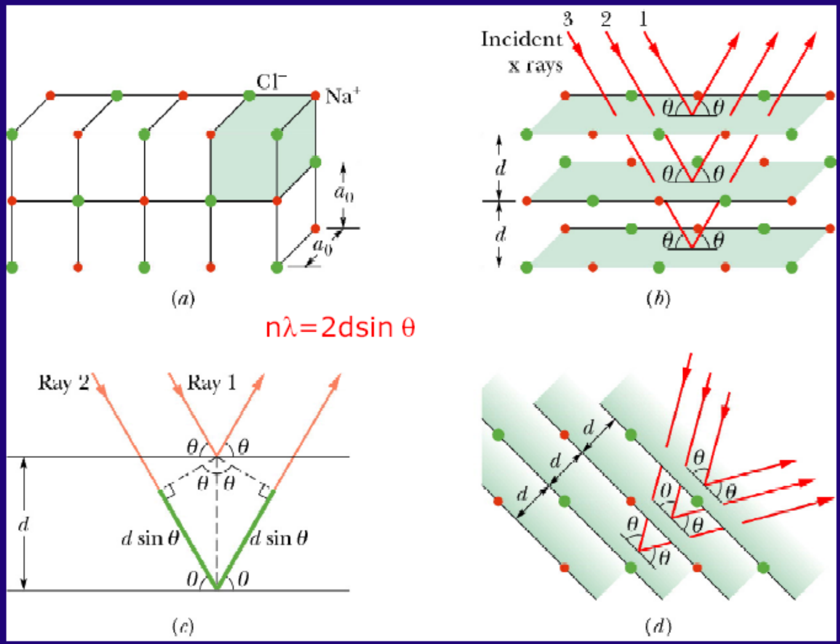
Interference occurs when the path difference between scatterings from different layers is an integral number of wavelengths.



- Since the momentum p = mv of the electrons can be measured, the wavelength predicted by de Broglie is known - $\lambda = h / p = h / mv$. One can test for interference by changing the speed (i.e. the kinetic energy mv^2) of the electrons.

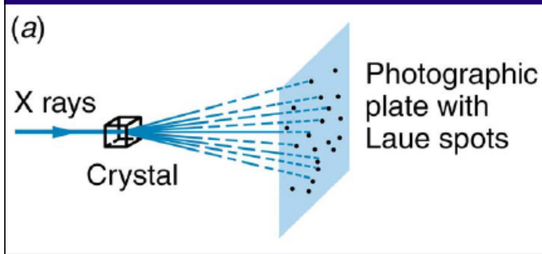
•Result: Electrons show interference like waves with wavelength $\lambda = h / p$

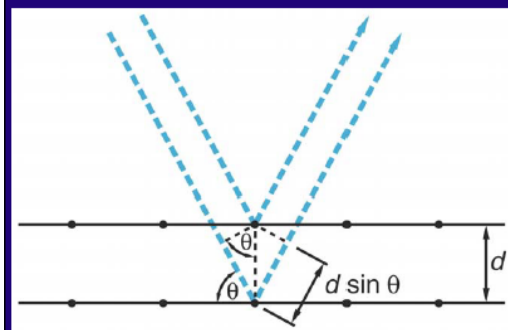
NaCl & X-ray Diffractio : Orientation Important!

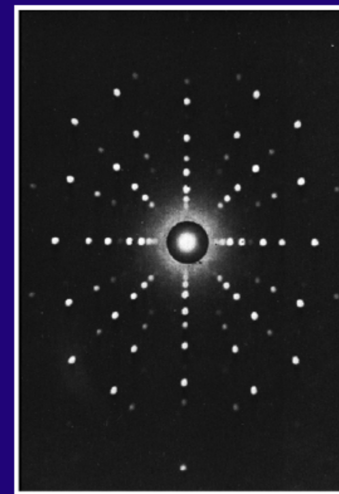


$n\lambda = 2d\sin\theta$

Bragg Scattering of X-Ray Light





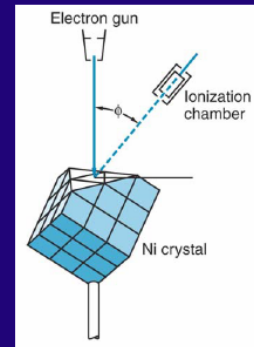


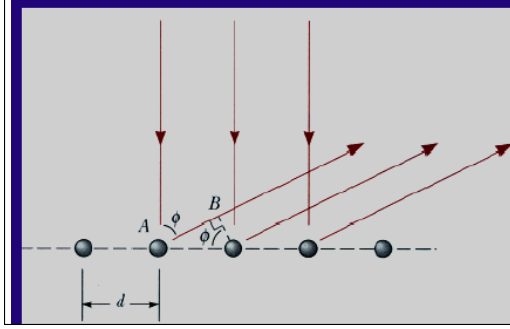
Electron Diffraction : Davisson Germer Expt

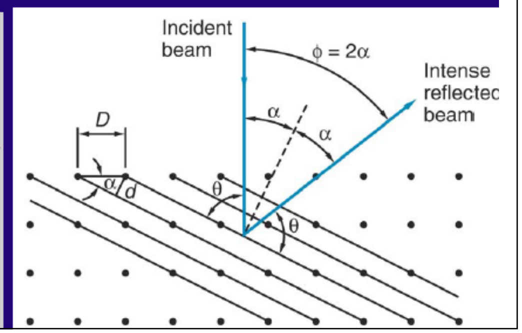
Matter Waves

de Broglie Conjecture

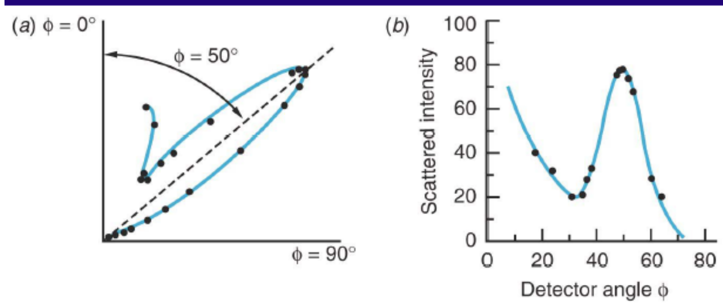
$$\lambda = \frac{h}{p}$$







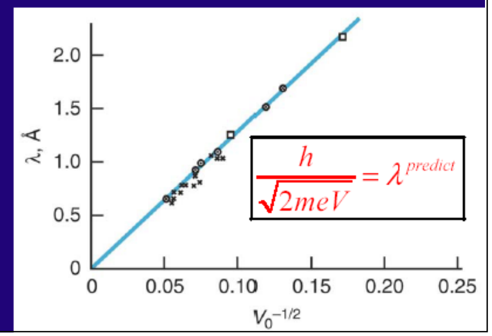
Electron Diffraction : Davisson Germer Expt



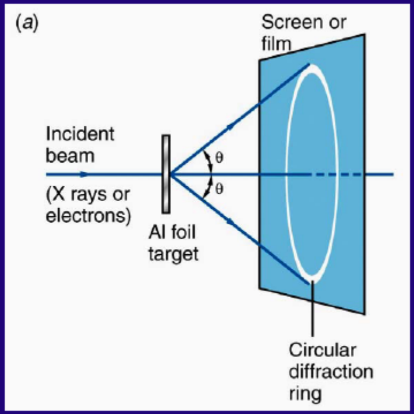
$$\frac{1}{2}mv^2 = K = \frac{p^2}{2m} = eV$$

$$\Rightarrow v = \sqrt{\frac{2eV}{m}} \quad ;$$

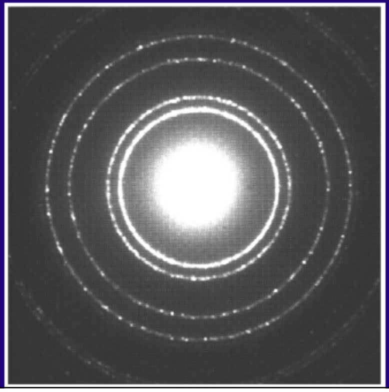
$$p = mv = m\sqrt{\frac{2eV}{m}}$$



Diffraction Pattern in Polycrystalline Al target

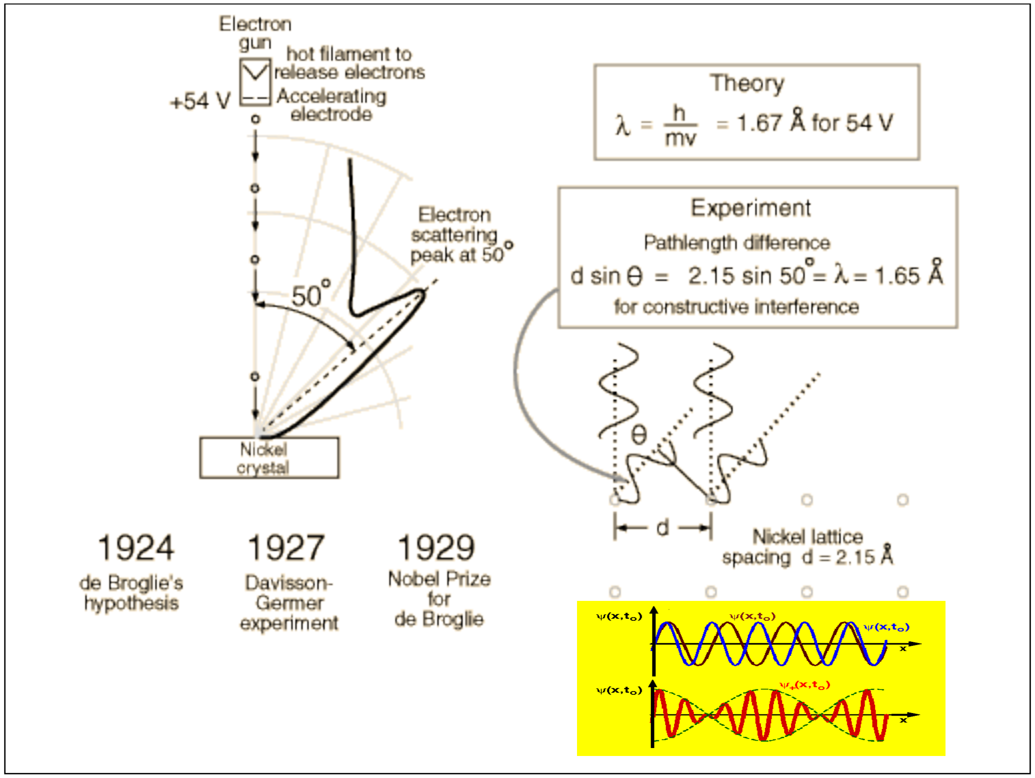


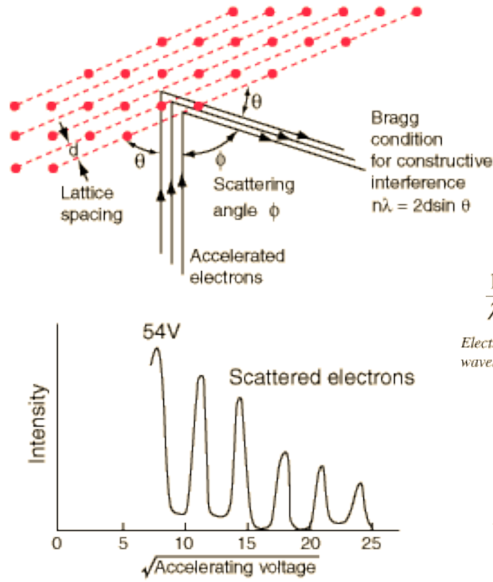
Diffraction pattern produced by 600eV electrons incident on a Al foil target



e- diffraction and the Davisson-Germer Experiment

- Applet


Electron gun
hot filament to release electrons
Accelerating electrode
+54 V
Electron scattering peak at 50°
50°
Nickel crystal
Theory
λ = h/mv = 1.67 Å for 54 V
Experiment
Pathlength difference
d sin θ = 2.15 sin 50° = λ = 1.65 Å
for constructive interference
θ
d
Nickel lattice spacing d = 2.15 Å
1924
de Broglie's hypothesis
1927
Davisson-Germer experiment
1929
Nobel Prize for de Broglie



$$\frac{1}{\lambda} = \frac{n}{2d\sin\theta} = \frac{p}{h} = \frac{\sqrt{2mE}}{h} = \frac{\sqrt{2meV}}{h}$$

Electron wavelength — *Bragg law* — *deBroglie relationship* — *Acceleration through voltage V*

$$\frac{1}{\lambda(nm)} = \frac{n}{2d\sin\theta} = 0.815\sqrt{V}$$

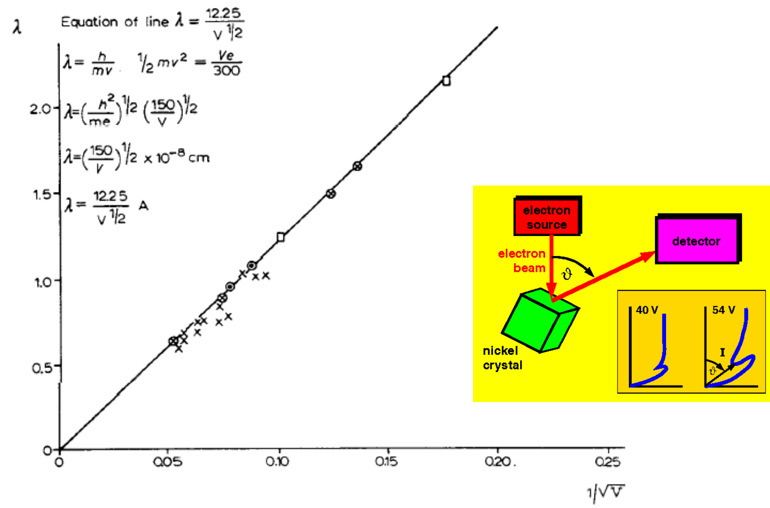


Fig. 4. Test of the de Broglie formula $\lambda = k/p = h/mv$. Wavelength computed from diffraction data plotted against $1/V^{1/2}$, (V, primary-beam voltage). For precise verification of the formula all points should fall on the line $\lambda, = 12.25/V^{1/2}$ plotted in the diagram. (x From observations with diffraction apparatus; o same, particularly reliable; □ same, grazing beams. ⊙ From observations with reflection apparatus.)

Notes

- Before you come to lab you must read the lab manual and identify the warnings/ cautions in the lab manual.
- **Mark the warnings/cautions in RED and have your TA verify before starting the experiment.**

e/m for the electron

Determination of e/m for Electron

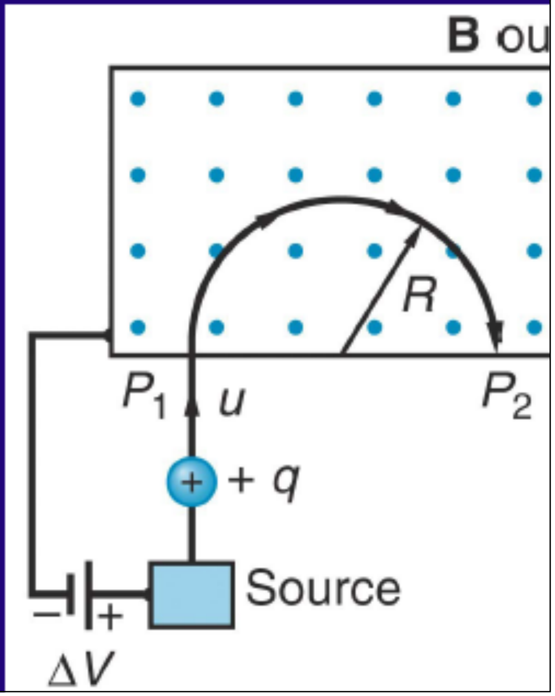
e/m is characteristic of a particle : electron Vs Cl^- ion

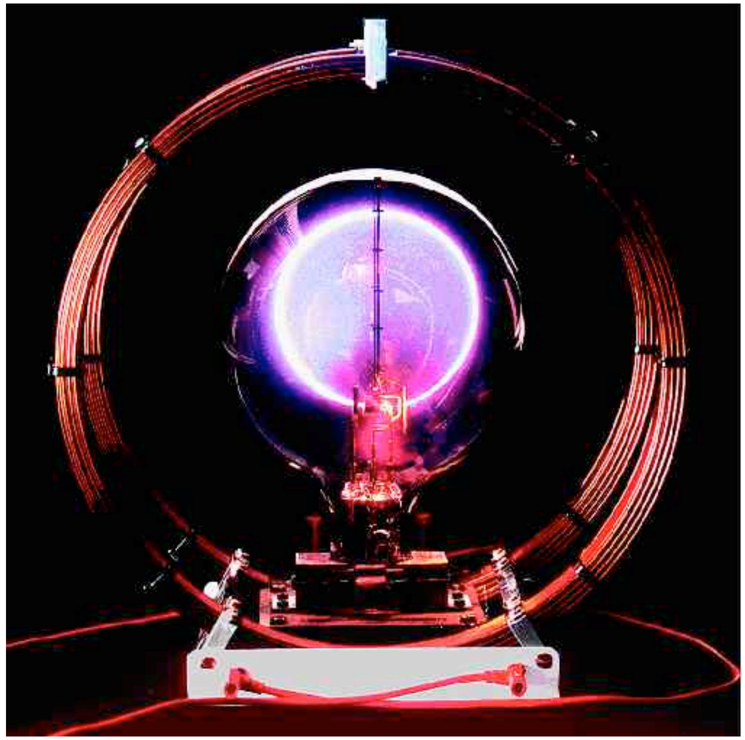
When Uniform magnetic field of strength B is established perpendicular to direction of motion of a charged particle, particle moves in a circular path of radius R

$$uB = \frac{mu^2}{R} \Rightarrow R = \frac{mu}{qB}$$

f electrons have KE = qV

hen $\frac{e}{m} = \frac{2V}{B^2 R^2}$



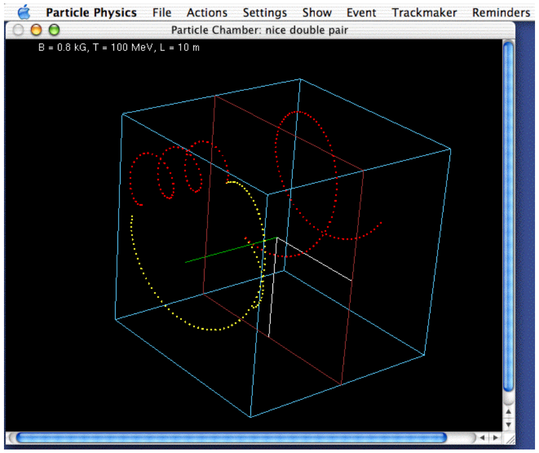


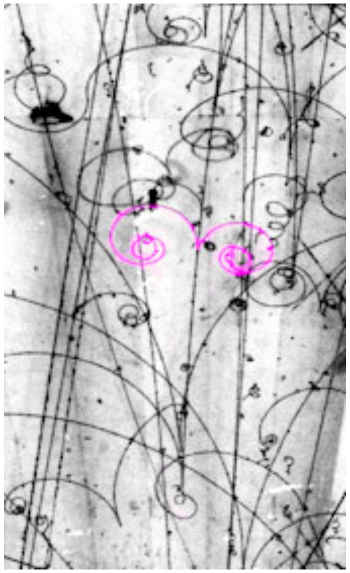
e/m, e, m of Electron : Why Important

Realization that electron is much less massive than the Hydrogen atom made physicists think about the structure Inside atom

The electron was discovered just a bit over 100 years ago, triggered A scientific revolution

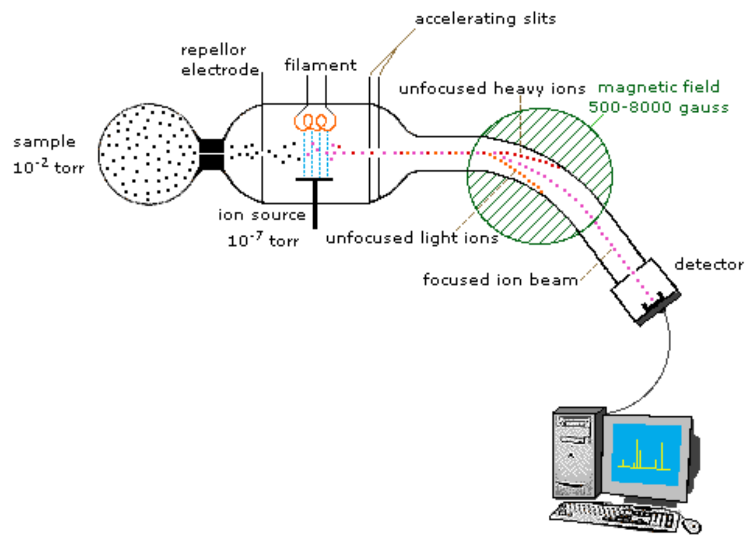
Thomson's idea
Still used to measure
Masses of fundamen
Particles or nuclei





Electron-Positron Pair

Mass Spectrometer



Optical Coherence Slides

2DL

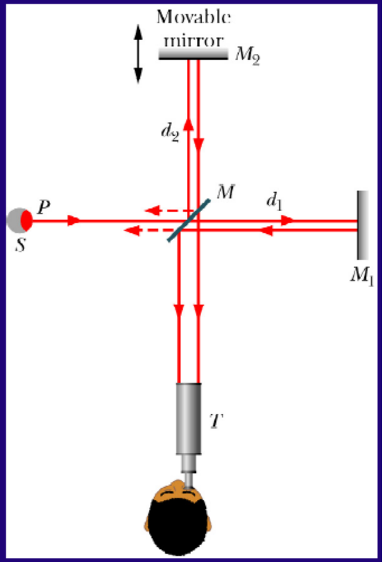
Michelson's Interferometer

Interferometer: device to measure lengths or changes in lengths with great accuracy by means of interference fringes (big daddy of them all was designed by Michelson in 1881…first American Nobel prize 1907)

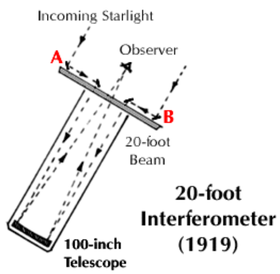
How it works:

- Light from source at P encounters beam splitter
- Beam splitter transmits ½ and reflects ½ of incident
- The 2 waves now head towards M1 and M2 mirrors
- Get reflected entirely and sent back along direction of incidence and then deflected towards telescope T
- Observer at T sees a pattern of "zebra strip" like fringes

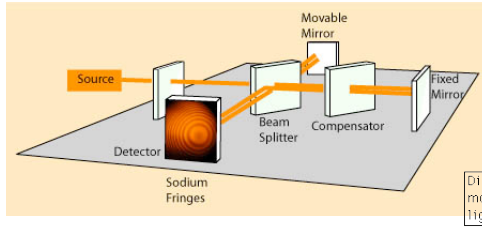
Path length Δ when 2 waves combine at telescope=$2d_2 - 2d_1$ anything that changes this path diff Δ will cause change in phase diff between two waves at the eye. E.g. If mirror M1 moves by $\lambda/2$ then Δ changes by λ and fringe pattern shifted by 1 (max→ min)

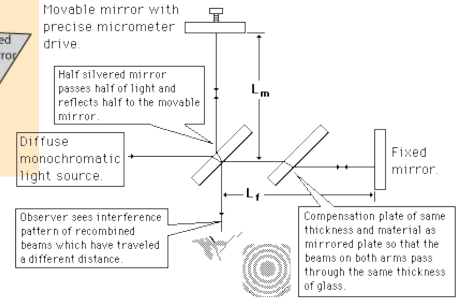


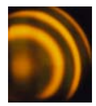
Stellar Interferometry



Interferometry







$$\ell_{coherence} = c \cdot t_{coherence}$$

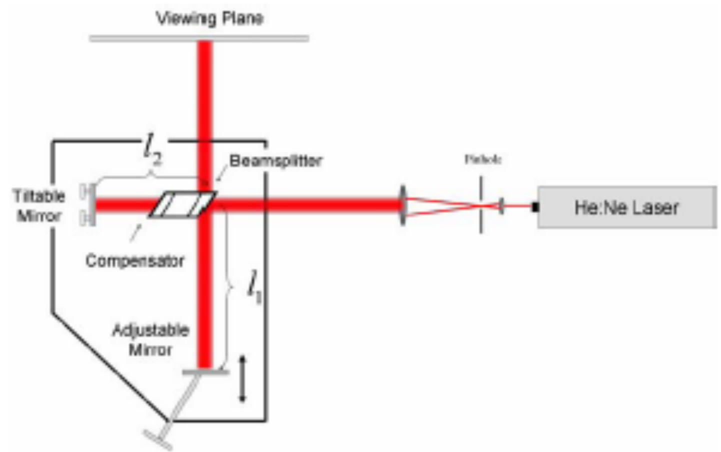
$$\begin{aligned}
\Delta\nu &= \nu_{\max} - \nu_{\min} \\
&= \frac{c}{\lambda_{\min}} - \frac{c}{\lambda_{\max}} = c \cdot \left(\frac{1}{\lambda_{\min}} - \frac{1}{\lambda_{\max}}\right) \\
&= c \cdot \left(\frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} \cdot \lambda_{\min}}\right) \\
&= \frac{c \cdot \Delta\lambda}{\lambda_{\max} \cdot \lambda_{\min}} \left[\frac{\text{cycles}}{\text{second}} = \text{Hz}\right]
\end{aligned}$$

For a laser, $\lambda_{\max} \simeq \lambda_{\min} \simeq \lambda_0$ and $\Delta\lambda \simeq 0$, so the temporal bandwidth is very small:

$$\Delta\nu \simeq \frac{c \cdot 0}{\lambda_0^2} \to 0$$

The reciprocal of the temporal bandwidth has dimensions of time and is the *coherence time*.

$$\begin{aligned}
t_{coherence} &= \frac{1}{\Delta\nu}\,[s] \to \infty \text{ for laser} \\
\ell_{coherence} &= \frac{c}{\Delta\nu}\,[\text{m}] = \frac{\lambda_{\max} \cdot \lambda_{\min}}{\Delta\lambda} \to \infty \text{ for laser}
\end{aligned}$$

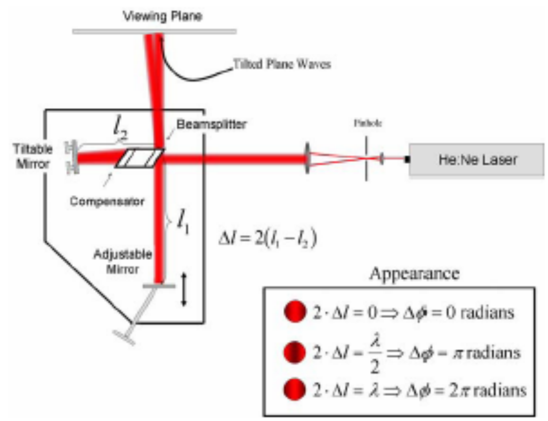


Michelson interferometer using He:Ne laser for illumination ($\lambda_0 = 632.8$ nm).

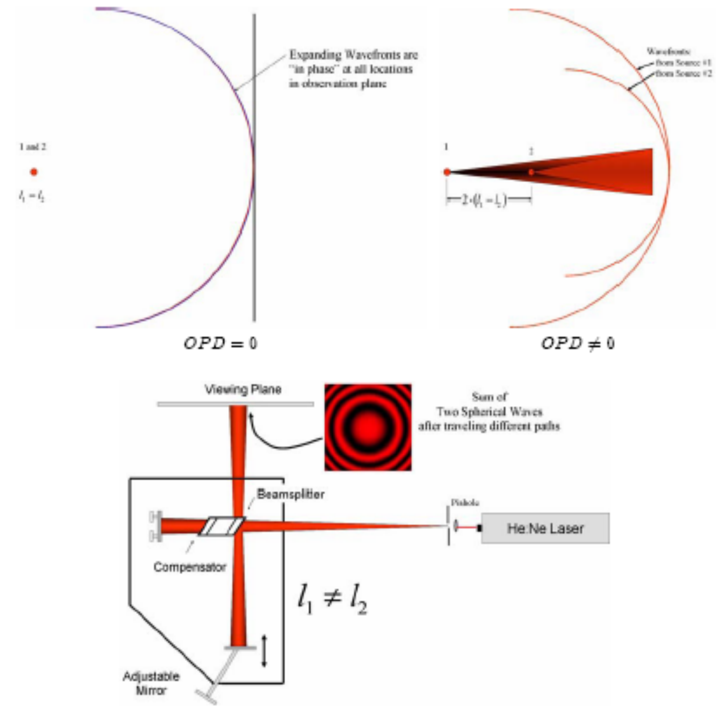
$$OPD = \frac{2 \cdot \Delta\ell}{\lambda_0} \quad \text{[wavelengths]}$$

$$OPD = 2 \cdot \ell_1 - 2 \cdot \ell_2 = 2 \cdot \Delta\ell \quad \text{[m]}$$

$$\begin{aligned} O\Phi D &= 2\pi \left[\frac{\text{radians}}{\text{wavelength}}\right] \cdot \frac{2 \cdot \Delta\ell}{\lambda_0} \quad \text{[wavelengths]} \\ &= 2\pi \cdot \frac{2 \cdot \Delta\ell}{\lambda_0} \quad \text{[radians]} \end{aligned}$$



Michelson interferometer with collimated light so that one beam is tilted relative to the other. The optical phase difference varies linearly across the field, producing linear fringes just like Young's two-slit interference.


Expanding Wavefronts are "in phase" at all locations in observation plane
1 and 2
$l_1 = l_2$
$OPD = 0$
Wavefronts:
from Source #1
from Source #2
1
2
$2 \cdot (l_1 - l_2)$
$OPD \neq 0$
Viewing Plane
Sum of Two Spherical Waves after traveling different paths
Beamsplitter
Pinhole
He:Ne Laser
Compensator
$l_1 \neq l_2$
Adjustable Mirror

Questions?

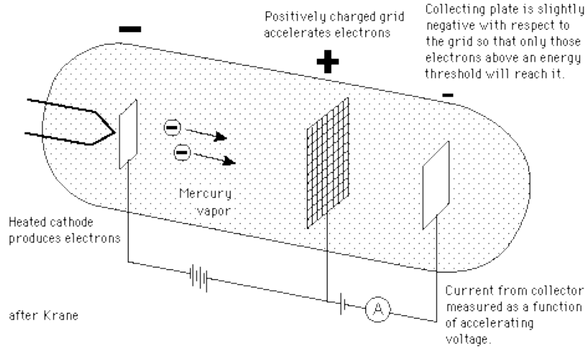
Explain the direction of motion of the circular fringes when the path length is changed, i.e., what directions do the circular fringes move if the OPL is increased? What if OPL is decreased?.

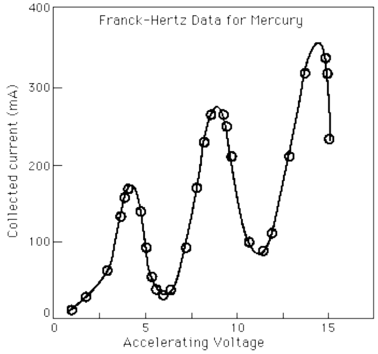
When the Michelson is used with collimated light, explain how a single dark fringe can be obtained. Where did the light intensity go?

Explain what happens when a piece of glass (or other material) is placed in one arm.

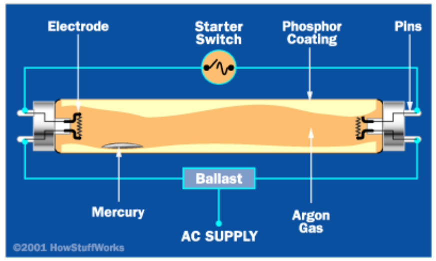
e- Collisions with matter and the Franck Hertz Experiment

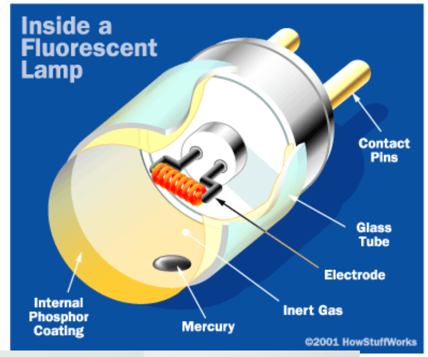
Franck-Hertz

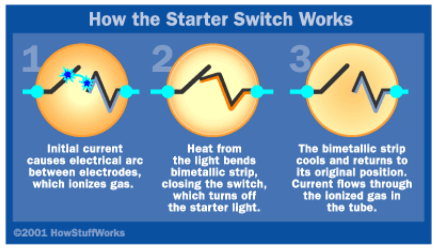




e^--Atom collisions overhead!

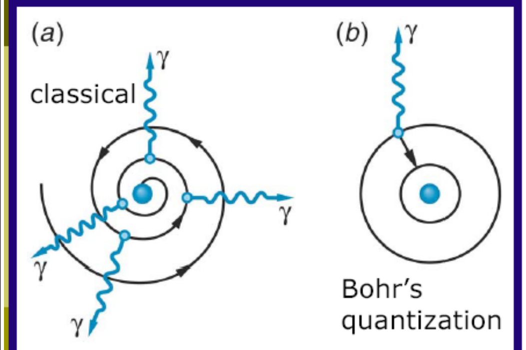


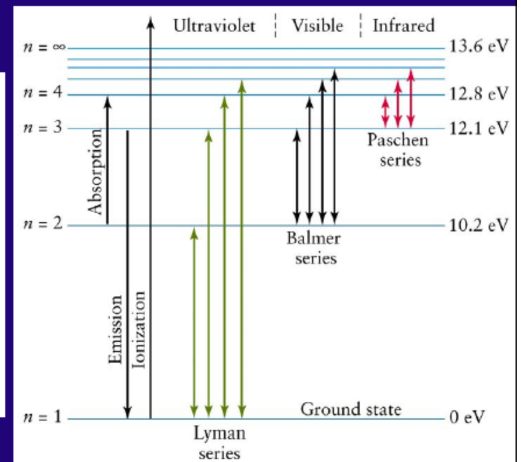




Franck-Hertz Experiment : A prelude

Bohr Atom : Discrete orbit → Emission & Absorption line



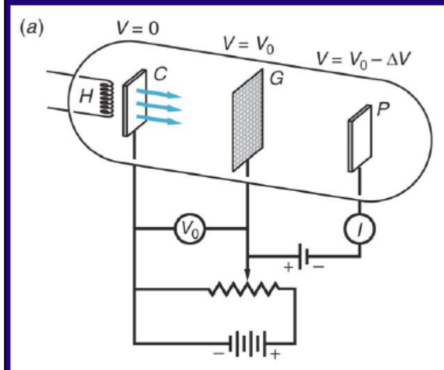


$$r_n = \frac{n^2\hbar^2}{mke^2},\ n = 1, 2, \ldots\infty$$

$n = 1 \Rightarrow$ Bohr Radius a_0

$$E_n = -\left(\frac{ke^2}{2a_0}\right)\frac{Z^2}{n^2}$$

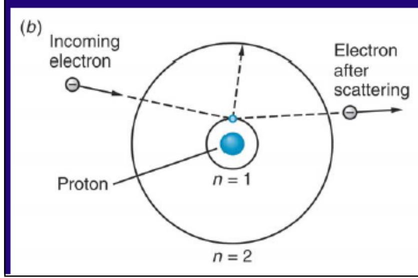
Franck Hertz Experiment: Playing Football !



Inelastic scattering of electrons
Confirms Bohr's Energy quantization

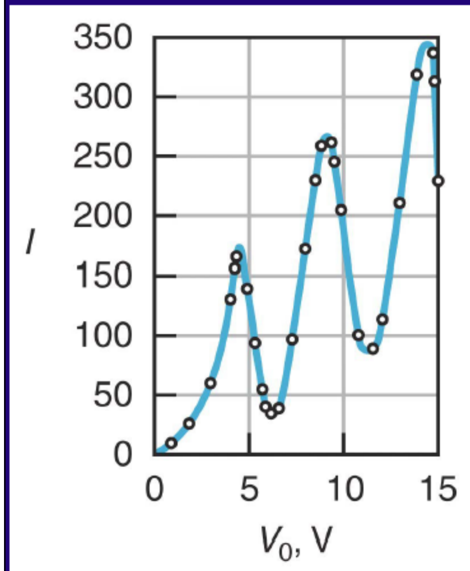
Electrons ejected from heated cathode
At zero potential are drawn towards
the positive grid G. Those passing thru
Hole in grid can reach plate P and cause
Current in circuit if they have sufficient
Kinetic energy to overcome the retarding
Potential between G and P

Tube contains low pressure gas of stuff!



If incoming electron does not have
enough energy to transfer $\Delta E = E_2 - E_1$ then
Elastic scattering, if electron has atleast
KE= ΔE then inelastic scattering and the
electron does not make it to the plate P
→ Loss of current

(J) Franck & (G) Hertz Experiment

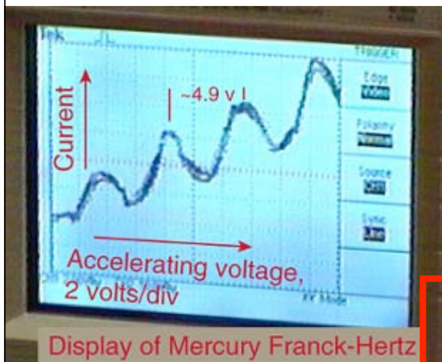


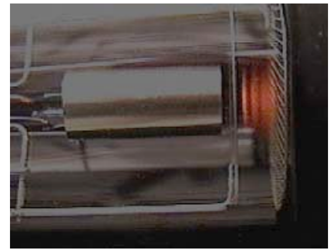
Current decreases because many
Electrons lose energy due to inelasti
Scattering with the Hg atom in tube
And therefore can not overcome the
Small retarding potential between
G → P

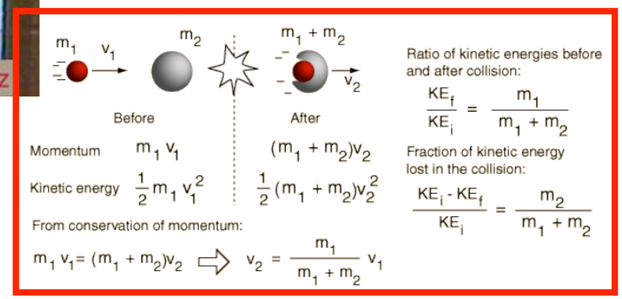
The regular spacing of the peaks
Indicates that ONLY a certain quanti
Of energy can be lost to the Hg ato
ΔE=4.9 eV.

This interpretation can be confirmed
Observation of radiation of photon e
E=hf=4.9 eV emitted by Hg atom wl
V_0 > 4.9V

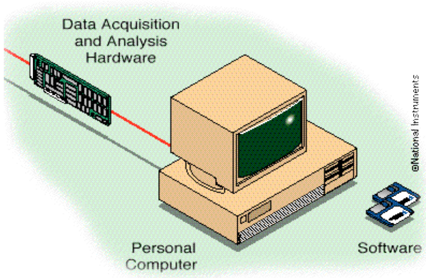
Electrons are accelerated in the Franck-Hertz apparatus and the collected current rises with accelerated voltage. As the Franck-Hertz data shows, when the accelerating voltage reaches 4.9 volts, the current sharply drops, indicating the sharp onset of a new phenomenon which takes enough energy away from the electrons that they cannot reach the collector. This drop is attributed to inelastic collisions between the accelerated electrons and atomic electrons in the mercury atoms. The sudden onset suggests that the mercury electrons cannot accept energy until it reaches the threshold for elevating them to an excited state. This 4.9 volt excited state corresponds to a strong line in the ultraviolet emission spectrum of mercury at 254 nm (a 4.9eV photon). Drops in the collected current occur at multiples of 4.9 volts since an accelerated electron which has 4.9 eV of energy removed in a collision can be re-accelerated to produce other such collisions at multiples of 4.9 volts. This experiment was strong confirmation of the idea of quantized atomic energy levels.

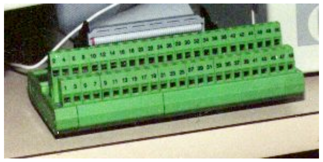


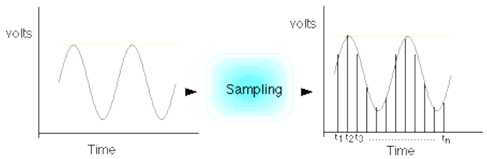


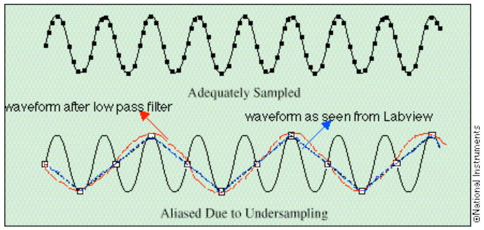


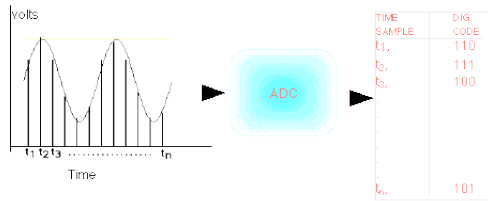
Electronic Measurement using Digital to Analog Conversion

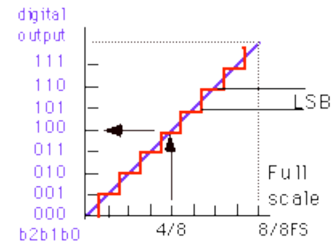


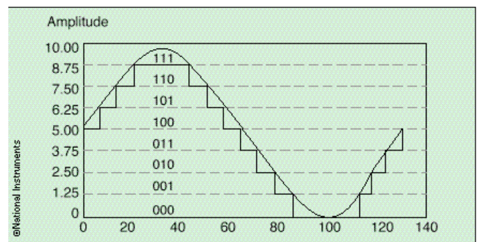







volts
t1 t2 t3
tn
Time
ADC
TIME SAMPLE
DIG CODE
t1. 110
t2. 111
t3. 100
tn. 101


digital output
111
110
101
100
011
010
001
000
b2b1b0
LSB
Full scale
4/8
8/8FS


Amplitude
10.00
8.75
7.50
6.25
5.00
3.75
2.50
1.25
0
111
110
101
100
011
010
001
000
0 20 40 60 80 100 120 140
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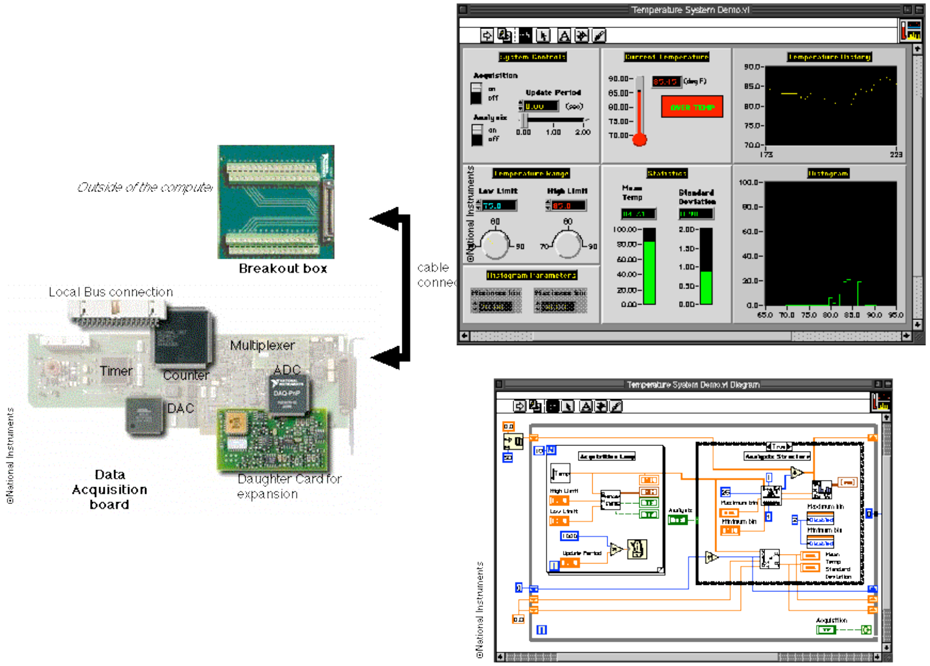
Outside of the compute
Breakout box
cable
Local Bus connection
Multiplexer
Timer
Counter
ADC
DAC
Data Acquisition board
Daughter Card for expansion
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Temperature System Demo.vi
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Temperature System Demo.vi Diagram
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